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SOLUTION OF THE MHD BOUNDARY LAYER FLOW MOVING CONTINUOUS FLAT SURFACE USING SPLINE METHOD

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ABSTRACT

Here we solve the equation of Magneto hydrodynamic boundary layer flows past a moving continuous flat surface in the presence of transverse magnetic field. Here we check effect of magnetic field on velocity of MHD fluid. Solve the governing nonlinear equations with use their associated boundary conditions with blue method. The beauty of this method is we can solve nonlinear problem directly, without convert in linear form. Numerical solution and graphical presentation of flow problem is also given.

NOMENCLATURE

MHD - Magneto hydrodynamics

u, v - Velocity components in X, Y directions respectively

U, W - The mainstream velocity components

ρ - Field density

ψ - Stream function

σ - Electrical conductivity

$B_y(x)$ -magnetic field strength

v_m - Magnetic viscosity

S - Magnetic field strength

η - The single independent variable

KEYWORDS: Boundary Layers, Blue Method, Linear Equations, Magnetic Field, Third Order Nonlinear Differential Equation

1. INTRODUCTION

Due to wide application in several technical and industrial processes, the boundary layer flow over continuous moving surface have attracted many researcher in many branches of engineering. MHD is the motion of an electrically conducting fluid in the presence of a magnetic field. Due to the motion of an electrically conducting fluid in a magnetic field the electrical currents produces their own magnetic field and these modify the original magnetic field. This induced current interacts with the magnetic field to produce electromagnetic forces effects the original motion. Thus the two important basic effects of Magneto hydrodynamics are (i) the motion of the fluid affects the magnetic field and (ii) the magnetic field affects the motion of the fluid. MHD flow over moving surfaces emerges in a large variety of industrial and technological applications. It has been investigated by many researchers. [1] Wu (1973) has studied the effects of suction or injection on MHD boundary layer flow. [2] Takhar et.al. (1987) studied a MHD asymmetric flow over a semi-infinite moving surface [3] Mahapatra and Gupta (2001) studied a steady two- dimensional stagnation-point flow by a uniform transverse magnetic field. [4] Jean-David Hoernel (2008) has been investigated the steady laminar incompressible boundary layer governing MHD flow. [5] Rajput (2013) studied of MHD boundary layer flow. [6] Sharma Vineetkumar (2011), Numerical solution of two dimensional MHD stagnation-point flow. [7] Patel M. (2013) had discussed group theoretical method. [8] Pandya Jigisha et al. (2010) had studied föppl-hencky equation using spline method.

The governing equation is nonlinear differential equations, which is solved by using spline collocation blue method. In this way, the paper has been organized as follows. In section 2, basic equations for given problem and in Section 3, spline collocation blue method and approximate solution for the governing equations. The results and discussion are in section 4.Conclusions are summarized in section 5.

2. BASIC EQUATIONS

[9]Srivastava et al (1987), the basic equations governing the motion of two dimensional, steady incompressible viscous fluids past continuous surface in the presence of transverse magnetic field can be written as follow:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = v_m \frac{\partial^3 \psi}{\partial y^3} - S(x)\frac{\partial \psi}{\partial y} \tag{2.1}$$

Where $S(x) = \dfrac{\sigma B_y^2(x)}{\rho}$

With the boundary conditions:

$$\frac{\partial \psi}{\partial y} = U(x), \frac{\partial \psi}{\partial x} = -V(x) \text{ at } y = 0 \tag{2.2}$$

$$\frac{\partial \psi}{\partial y} = 0 \text{ at } y \to \infty \tag{2.3}$$

Where x co-ordinate measured along the plate, y co-ordinate normal to the plate, u and v are the velocity component of the fluid in X and Y directions. v_m - magnetic viscosity. ρ - fluid density, σ - electrical conductivity, $B_y(x)$ -magnetic field strength along the plat wall and acting perpendicular to it, ψ - stream function.

To study the physical phenomena of flow under consideration, solve above equations (2.1), (2.2), (2.3). Reduce the partial differential equations into ordinary differential equations. Patel M. [7] reduces the partial differential equation into ordinary differential equation. For that they employ following one parameter linear group transformation Γ_1.

Here Γ_1 is selected as (6)

$$
\begin{aligned}
&x = A^{\alpha_1}\overline{x}, \quad \mathrm{y} = A^{\alpha_2}\overline{x} \\
&\Gamma_1 : \psi = A^{\alpha_3}\overline{\psi}, \quad \mathrm{S}(x) = A^{\alpha_4}\overline{S}(x) \\
&U = A^{\alpha_5}\overline{U}, \qquad V = A^{\alpha_6}\overline{V}
\end{aligned}
$$

Where $\alpha_i : i = 1.....6$ and A are real arbitrary constant.

Now seek relations $\alpha's$ such that the basic equations (1) along with the boundary conditions (2) and (3) will be invariant under this group transformation Γ_1.

Invariant conditions demand that the power of A in each term of transformed equations should be equal, which yields

$$2\alpha_3 - 2\alpha_2 - \alpha_1 = \alpha_3 - 3\alpha_2 = \alpha_4 + \alpha_3 - \alpha_2 \tag{2.4}$$

$$\alpha_3 - \alpha_2 = \alpha_5 \tag{2.5}$$

$$\alpha_3 - \alpha_1 = \alpha_6 \tag{2.6}$$

Solving (2.4), (2.5) and (2.6) we get

$$\frac{\alpha_2}{\alpha_1} = m(\text{say}), \quad \frac{\alpha_3}{\alpha_1} = 1 - m, \quad \frac{\alpha_4}{\alpha_1} = -2m, \quad \frac{\alpha_5}{\alpha_1} = 1 - 2m, \quad \frac{\alpha_6}{\alpha_1} = -m \tag{2.7}$$

3. QUARTIC SPLINE BLUE METHOD

For three points boundary value problems are

Let $s_i(x)$ be quartic spline in $\left[x_{i-1}, x_i\right]$. Conditions for natural splines are

i. $s_i(x)$ Almost quartic in each subinterval $\left[x_{i-1}, x_i\right]$.

ii. $s_i(x_i) = y_i$, for i = 0,1,2,........,n.

iii. $s_i(x_i)$, $\mathrm{s}_i^{'}(x_i)$, $\mathrm{s}_i^{''}(x_i)$, $\mathrm{s}^{'''}(x_i)$ are continuous in $\left[x_{0,} x_n\right]$.

iv. $s_i^{'''}(x_0) = s_i^{'''}(x_n) = 0.$

Here spline third derivative must be linear in $\left[x_{i-1}, x_i\right]$.

So $s_i^{'''}(x_i) = \dfrac{1}{h_i}\left[\left(x_i - x\right) y_{i-1}^{'''} + \left(x - x_{i-1}\right) y_i^{'''}\right]$ **(3.1)**

Where $h_i = x_i - x_{i-1}$ and $s'''(x_i) = y_i'''$

Integrate (1), twice with respect to x.

$$s_i'(x) = \frac{1}{h_i}\left[\frac{(x_i - x)^3}{6} y_{i-1}''' + \frac{(x - x_{i-1})^3}{6} y_i'''\right] + c_i(x_i - x) + d_i(x - x_{i-1}).$$

Where use $s_i'(x_{i-1}) = y_{i-1}'$ and $s_i'(x_i) = y_i'$ in (1).

We get constants c_i and d_i

$$c_i = \frac{1}{h_i}\left(y_{i-1}' - \frac{h_i^2}{6} y_{i-1}'''\right) \text{ and } d_i = \frac{1}{h_i}\left(y_i' - \frac{h_i^2}{6} y_i'''\right)$$

So

$$s_i'(x) = \frac{1}{h_i}\left[\frac{(x_i - x)^3}{6} y_{i-1}''' + \frac{(x - x_{i-1})^3}{6} y_i'''\right] + \frac{1}{h_i}\left(y_{i-1}' - \frac{h_i^2}{6} y_{i-1}'''\right)(x_i - x) + \frac{1}{h_i}\left(y_i' - \frac{h_i^2}{6} yi'''\right)(x - x_{i-1}). \tag{3.2}$$

Integrate (3.2), once with respect to x.

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i - x)^4}{24} y_{i-1}''' + \frac{(x - x_{i-1})^4}{24} y_i'''\right] - \frac{1}{h_i}\left(y_{i-1}' - \frac{h_i^2}{6} y_{i-1}'''\right)\frac{(x_i - x)^2}{2} + \frac{1}{h_i}\left(y_i' - \frac{h_i^2}{6} yi'''\right)\frac{(x - x_{i-1})^2}{2} + e_i. \tag{3.3}$$

Take $s_i(x_{i-1}) = y_{i-1}$, we get constants e_i

Where $e_i = y_{i-1} - \frac{h^3}{8} y_{i-1}''' + \frac{h}{2} y_{i-1}'$.

Substitute e_i in (3.3), we get

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i - x)^4}{24} y_{i-1}''' + \frac{(x - x_{i-1})^4}{24} y_i'''\right] - \frac{1}{h_i}\left(y_{i-1}' - \frac{h_i^2}{6} y_{i-1}'''\right)\frac{(x_i - x)^2}{2} + \frac{1}{h_i}\left(y_i' - \frac{h_i^2}{6} yi'''\right)\frac{(x - x_{i-1})^2}{2} + y_{i-1} - \frac{h^3}{8} y_{i-1}''' + \frac{h}{2} y_{i-1}'. \tag{3.4}$$

Here $s_i^{''}(x_i^-) = s_{i+1}^{''}(x_i^+)$ and for equal intervals we have,

$$y_{i+1}^{'} - 2y_i^{'} + y_{i-1}^{'} = \frac{h^2}{6}(y_{i+1}^{'''} + 4y_i^{'''} + y_{i-1}^{'''}) \tag{3.5}$$

And for $s_i(x_i^-) = s_{i+1}(x_i^+)$ and for equal intervals we have,

$$y_{i-1} - y_i = -\frac{h}{2}\left(y_i^{'} + y_{i-1}^{'}\right) + \frac{h^3}{24}(3y_{i-1}^{'''} - y_i^{'''}) \tag{3.6}$$

3.1. Solution by Using Collocation Method

[8] Srivastava et al (1987), the basic equations governing the motion of two dimensional, steady incompressible viscous fluids past continuous surface in the presence of transverse magnetic field can be written in non-linear equation as follow:

$$f^{'''} - \beta f^{'} + ff^{''} = 0, \tag{3.1.1}$$

Subject to boundary conditions as,

$$f^{'}(0) = 1,\ f(0) = 1,\ f^{'}(0.5) = 0. \tag{3.1.2}$$

To obtain the spline solution, we begin with a assume function $f(\eta) = -\eta^2 + \eta + 1$ which satisfy given boundary conditions (3.1.2). To find the solution of equation (3.1.1) along with boundary conditions (3.1.2). First we use $f(\eta) = -\eta^2 + \eta + 1$ and (3.1.1) in (3.5) and $h = 0.1$, we gate different values of $y_i^{'}$ for $i = 1, 2, 3, 4.$

To find the final solution we use (3.6) for different values of $i = 1, 2, 3, 4$ respectively

We get the equations as follows

$$\begin{aligned}
y_0 - y_1 &= -\frac{h}{2}[y_1^{'} + y_0^{'}] + \frac{h^3}{24}[-y_1^{'''} + 3y_0^{'''}] \\
y_1 - y_2 &= -\frac{h}{2}[y_2^{'} + y_1^{'}] + \frac{h^3}{24}[-y_2^{'''} + 3y_1^{'''}] \\
y_2 - y_3 &= -\frac{h}{2}[y_3^{'} + y_2^{'}] + \frac{h^3}{24}[-y_3^{'''} + 3y_2^{'''}] \\
y_3 - y_4 &= -\frac{h}{2}[y_4^{'} + y_3^{'}] + \frac{h^3}{24}[-y_4^{'''} + 3y_3^{'''}]
\end{aligned} \tag{3.1.3}$$

To substitute $y_i^{'}$ and $y_i^{'''}$ for $h = 0.1$ in (3.1.3). We get four unknown and four equations. Solve that equations using Matlab.

Table 1: Solution of Problem using Numerical Method

	Numerical Solution with Spline		
η	$f(\eta)$ **for** $\beta = 0$	$f(\eta)$ **for** $\beta = 1$	$f(\eta)$ **for** $\beta = 2$
0.0	1.0000000000	1.0000000000	1.0000000000
0.1	1.0875475000	1.0868558333	1.0861641666
0.2	1.1515783333	1.1494116666	1.1472450000
0.3	1.1943391666	1.1906141666	1.1868891666
0.4	1.2181966666	1.2133299999	1.2084633333
0.5	1.2255974999	1.2203058333	1.2150141667

Graphical Solution of Given Problem

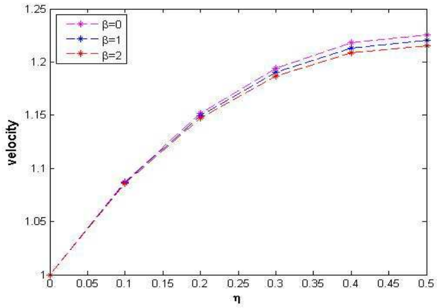

Figure 1: Velocity Profile Versus η for Different Values of β

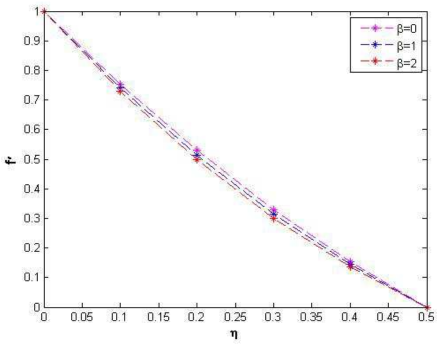

Figure 2: $f'(\eta)$ Versus η for Different Values of β

4. RESULTS AND DISCUSSIONS

From Figure-(1), it shows that, there is a significant impact of magnetic field on the velocity profile of the flow. Here for different values of β , we find velocity profile. From Figure it is clear that Value of β increase the velocity of fluid decrease. It means that for electrically conducting fluid in a magnetic field, magnetic field increase, velocity of fluid flow decrease. Magnetic field is transversely proportional to velocity.

5. CONCLUSIONS

Here we find the generalization of blue method for third order problem and solved the problem using blue method. The beauty of this method is no need to convert nonlinear problem into linear, we can solve directly in nonlinear form. This shows that spline method also gives nearest and accurate results. This shows the reliability of the method. Thus we can solve such type of problems using blue method without convert nonlinear problem into linear form.

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